Convergents

`Convergents[`*list*`]`
gives a list of the convergents corresponding to the continued fraction terms *list*.

`Convergents[`*x*, *n*`]`
gives the first *n* convergents for a number *x*.

`Convergents[`*x*`]`
gives if possible all convergents leading to the number *x*.

MORE INFORMATION

The convergents of the continued fraction $a_1+1/(a_2+1/(a_3+\ldots))$ are the rationals a_1, a_1+1/a_2, $a_1+1/(a_2+1/a_3)$, ….

For exact numbers, `Convergents[`*x*`]` can be used if *x* is rational or a quadratic irrational.

If *x* is a quadratic irrational or a representation of a quadratic irrational as a continued fraction, the final list element returned by `Convergents[`*x*`]` is the quadratic irrational represented by *x*.

For inexact numbers, `Convergents[`*x*`]` generates a list of all convergents that can be obtained given the precision of *x*.

`Convergents[`*x*, *n*`]` will return *n* convergents if possible. If *x* represents a rational or an inexact number, fewer than *n* terms may be returned.

EXAMPLES

CLOSE ALL

Basic Examples (3)

Generate the first 10 convergents to the Golden Ratio:

In[1]:= `Convergents[GoldenRatio, 10]`

Out[1]= $\left\{1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}\right\}$

In[1]:= **ContinuedFraction[Pi, 10]**

Out[1]= {3, 7, 15, 1, 292, 1, 1, 1, 2, 1}

In[2]:= **Convergents[%]**

Out[2]= $\left\{3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}\right\}$

In[3]:= **Convergents[Pi, 10]**

Out[3]= $\left\{3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}\right\}$

Quadratic irrationals have periodic continued fractions:

In[1]:= **ContinuedFraction[Sqrt[7]]**

Out[1]= {2, {1, 1, 1, 4}}

In[2]:= **Convergents[%]**

Out[2]= $\left\{2, 3, \frac{5}{2}, \frac{8}{3}, \sqrt{7}\right\}$

In[3]:= **Convergents[Sqrt[7], 10]**

Out[3]= $\left\{2, 3, \frac{5}{2}, \frac{8}{3}, \frac{37}{14}, \frac{45}{17}, \frac{82}{31}, \frac{127}{48}, \frac{590}{223}, \frac{717}{271}\right\}$

Give all convergents for a rational number:

In[1]:= **Convergents[7 / 17]**

Out[1]= $\left\{0, \frac{1}{2}, \frac{2}{5}, \frac{7}{17}\right\}$

Convergents continues until the precision of the input is reached:

In[1]:= **Convergents[5.67567]**

Out[1]= $\left\{5, 6, \frac{17}{3}, \frac{210}{37}, \frac{26897}{4739}, \frac{27107}{4776}, \frac{54004}{9515}, \frac{81111}{14291}, \frac{135115}{23806}, \frac{216226}{38097}\right\}$

Properties& Relations (2)

The convergents of a number converge to it while alternating sides:

In[1]:= **Convergents[π, 10] - N[π]**

Out[1]= $\{-0.141593, 0.00126449, -0.0000832196, 2.66764\times10^{-7}, -5.77891\times10^{-10}, 3.31628\times10^{-10}, -1.22356\times10^{-10}, 2.91434\times10^{-11}, -8.71525\times10^{-12}, 1.61071\times10^{-12}\}$

The results from Rationalize are not always among the list of convergents:

In[1]:= **Rationalize[Pi, 1 / 1000]**

Out[1]= $\frac{201}{64}$

In[2]:= **Convergents[Pi, 10]**

Out[2]= $\left\{3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}\right\}$

New in 6